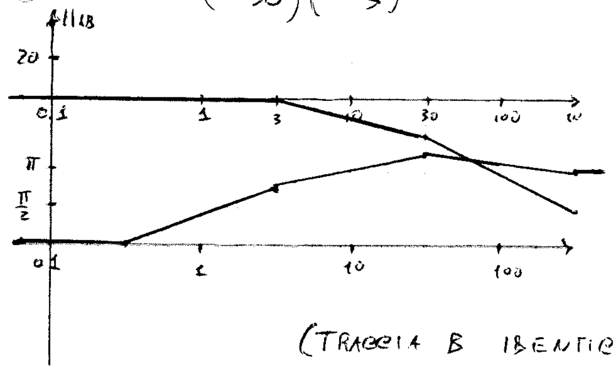
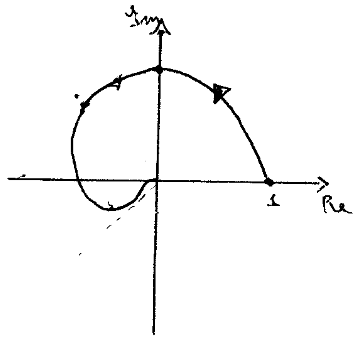
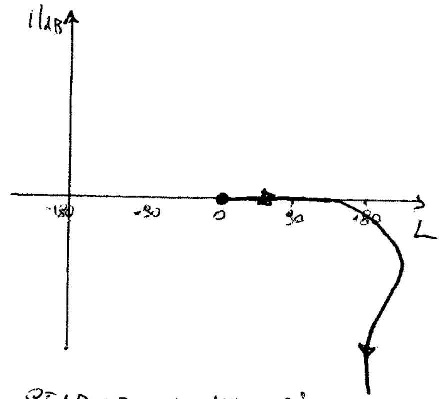
FONDAMENTI DI AUTOMATICA – 15/7/2008

1) $G(s) = \dfrac{1+\frac{s}{3}}{\left(1+\frac{s}{30}\right)\left(1-\frac{s}{3}\right)^2}$







NON ASINTOTICAMENTE STABILE $\Rightarrow$ NON E' UN FILTRO

(TRACCIA B IDENTICA, CAMBIANO SOLO I PUNTI DI ROTTURA: 5 e 50)

2) a) $S1: \begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y_1 = \begin{pmatrix} 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{cases}$ $\quad S2: \begin{cases} \dot{x}_3 = -\frac{1}{2} x_3 + u_2 \\ y = 2x_3 \end{cases}$

$u_2 = y_1$

$\Rightarrow \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 4 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} u$

$y = \begin{pmatrix} 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

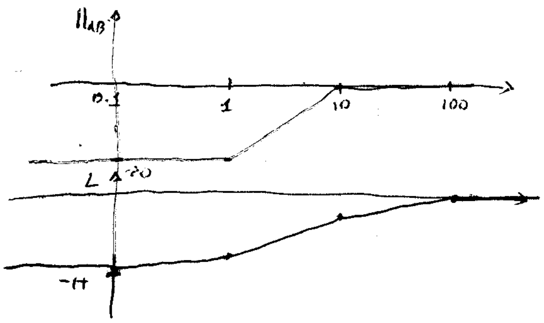
b) $G = \dfrac{16s}{(s^2+s+1)(1+2s)}$ $\quad y = G(0)\cdot 3 + 3\cdot |G(j1)| \sin(t + 0.2 + \angle G(j1))$

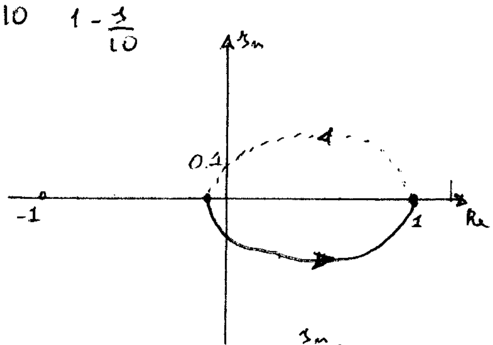
$G(0) = \emptyset \quad |G(j1)| = 7.16 \quad \angle G(j1) = -1.11$

c) $t > 5 \quad x_3(5) = \dfrac{y(5)}{2} = \dfrac{1.93}{2} \quad y(t) = 2\cdot x_3(5) \cdot e^{-\frac{1}{2}(t-5)}\, 1(t-5)$

(TRACCIA B: ANALOGO $\;y = 4.3, 16 \sin(t+0.3-1.11)$ $\quad y(5) = 5,41$)

3) $k>0$. Si sceglie $k=1$. $F(s) = -\dfrac{1}{10}\,\dfrac{1+s}{1-\frac{s}{10}}$

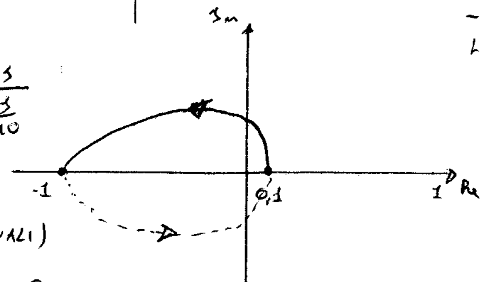




$\boxed{K>10}$

PER AVERE UN GIRO INTORNO A -1 E QUINDI LA STABILITA

$k<0$. Si sceglie $k=-1$. $F(s) = \dfrac{1}{10}\,\dfrac{1+s}{1-\frac{s}{10}}$



$\boxed{K<-1}$

(TRACCIA B: PROCEDIMENTO SIMILE, RISULTATI UGUALI)

4) a) $\begin{cases} \bar{x}_1^3 + \bar{x}_1 = 0 \\ \bar{x}_2 = \bar{x}_3 \\ \bar{x}_3 = \bar{x}_1 - \bar{x}_2 \end{cases} \rightarrow \begin{cases} \bar{x}_1(\bar{x}_1^2+1) = 0 \\ \bar{x}_2 = \bar{x}_3 \\ \bar{x}_3 = \bar{x}_1 - \bar{x}_2 \end{cases} \rightarrow \begin{cases} \bar{x}_1 = 0 \\ \bar{x}_2 = \bar{x}_3 \\ \bar{x}_3 = -\bar{x}_2 \end{cases} \Rightarrow \bar{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u(k)$ $\quad y(k) = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix}$

SEMPLICEMENTE STABILE (AUTOV: $0, j, -j$)